1/29/18 "Its always too early to quit" - Norman Peale

HW: "Inverses of Linear Functions" homework section

AIM: How do we find inverses of linear functions?

Warm Up:

Recall that functions have inverses that are also functions if they are one-to-one. With the exception of horizontal lines, all linear functions are one-to-one and thus have inverses that are also functions.

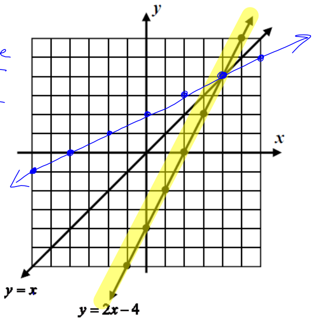
***Exercise* #1:** On the grid below the linear function $y = 2x - 4$ is graphed along with the line $y = x$.

$y=2x-4$

| x | y |
|---|---|
| -1 | -6 |
| 0 | -4 |
| 1 | -2 |
| 2 | 0 |
| 3 | 2 |
| 4 | 4 |
| 5 | 6 |
| 6 | 8 |

Inverse

| x | y |
|---|---|
| -6 | -1 |
| -4 | 0 |
| -2 | 1 |
| 0 | 2 |
| 2 | 3 |
| 4 | 4 |
| 6 | 5 |
| 8 | 6 |




(a) How can you quickly tell that $y = 2x - 4$ is a one-to-one function?

Use horizontal line test.

(b) Graph the inverse of $y = 2x - 4$ on the same grid. Recall that this is easily done by switching the $x$ and $y$ coordinates of the original line.

(c) What can be said about the graphs of $y = 2x - 4$ and its inverse with respect to the line $y = x$?

The graph and inverse are reflections over the line $y=x$.

$y=x$ is the line of symmetry between function and inverse.

(d) Find the equation of the inverse in $y = mx + b$ form.

$y=2x-4$

Inverse $x=2y-4$

$+4 \quad\quad +4$

$x+4=2y$

$y=\frac{x+4}{2}$

$y=\frac{x}{2}+\frac{4}{2}$

$\boxed{y=\frac{1}{2}x+2}$

(e) Find the equation of the inverse in $y = \dfrac{x+b}{a}$ form.

see above

As we can see from part (e) in *Exercise* #1, inverses of linear functions include the inverse operations of the original function but in reverse order. This gives rise to a simple method of finding the equation of any inverse. **Simply switch the $x$ and $y$ variables in the original equation and solve for $y$.**

***Exercise* #2:** Which of the following represents the equation of the inverse of $y = 5x - 20$?

(1) $y = -\frac{1}{5}x + 20$ (3) $y = \frac{1}{5}x - 4$

(2) $y = \frac{1}{5}x - 20$ (4) $y = \frac{1}{5}x + 4$

Inverse $+20$ then divide by 5

$x=5y-20$

$\frac{x+20}{5}=\frac{5y}{5}$

$\frac{x+20}{5}=y$

$\frac{1}{5}x+4$

Although this is a simple enough procedure, certain problems can lead to common errors when solving for $y$. Care should be taken with each algebraic step.

***Exercise* #3:** Which of the following represents the inverse of the linear function $y=\frac{2}{3}x+8$?

(1) $y=\frac{3}{2}x-8$ (3) $y=-\frac{3}{2}x+8$

(2) $y=\frac{3}{2}x-12$ (4) $y=-\frac{3}{2}x+12$

$x=\frac{2}{3}y+8$

$x-8=\frac{2}{3}y$

$\frac{3}{2}(x-8)=y$

$\frac{3}{2}x-12=y$

***Exercise* #4:** What is the $y$-intercept of the inverse of $y=\frac{3}{5}x-9$?

(1) $y=15$ (3) $y=9$

(2) $y=\frac{1}{9}$ (4) $y=-\frac{5}{3}$

$x=\frac{3}{5}y-9$

$x+9=\frac{3}{5}y$

$\frac{5}{3}(x+9)=y$

$y=\frac{5}{3}x+15$

Sometimes we are asked to work with linear functions in their point-slope form. The method of finding the inverse and plotting it, though, do not change just because the linear equation is written in a different form.

***Exercise* #5:** Which of the following would be an equation for the inverse of $y+6=4(x-2)$?

(1) $y-2=\frac{1}{4}(x+6)$ (3) $y-6=-4(x+2)$

(2) $y-2=-\frac{1}{4}(x+6)$ (4) $y+2=-4(x-6)$

$x+6=4(y-2)$

$\frac{1}{4}(x+6)=y-2$

***Exercise* #6:** Which of the following points lies on the graph of the inverse of $y-8=5(x+2)$? Explain your choice.

(1) $(8,-2)$ (3) $(-10,40)$

(2) $(-8,2)$ (4) $(-2,8)$

***Exercise* #7:** Which of the following linear functions would *not* have an inverse that is also a function? Explain how you made your choice.

(1) $y=x$ (3) $y=2$

(2) $2y=x$ (4) $y=5x-1$